Worksheet: Number Systems 1

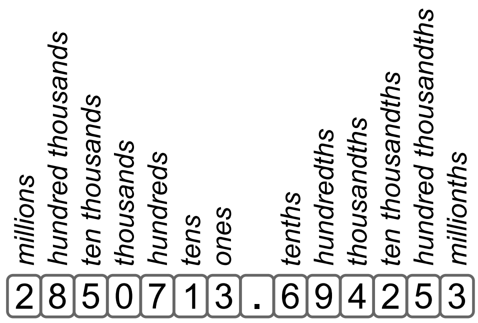
Denary Numbers

The base-10 number system represents numbers using ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The base-10 number system is most commonly referred to as the ***decimal system***; however, in materials from Pearson the term ***denary*** is used to refer to base-10 numbers. Therefore, I will try to use the word *denary* here.

The base-2 number system represents numbers using only two digits: 0 and 1. It is referred to as the *binary number system*, or simply the ***binary system***. Here we will review concepts you are familiar with in the denary system, and then extend those ideas so we may understand the binary system.

You will have surely learned in previous math courses that we can refer to the digits of a denary number by their *place value*. ***Place value*** is the concept that the value of a digit depends on its position in a number. In the base-10 system, each digit represents a power of 10. The diagram below and to the right shows the common names of the digits of a denary number.

This seems to be a good time to introduce another important concept. The ***most significant digit*** of a number, regardless of the number system, is always the left-most digit. In the diagram, this corresponds to the digit 2 that is in the millions place. It is most significant because increasing or decreasing the value of this digit affects the value of the overall number more significantly than changing the value of any other digit in the number. Similarly, the ***least significant digit*** is always the right-most digit. In the diagram, this corresponds to the digit 3 in the millionths place. Changing the value of this digit affects the value of the overall number less significantly than changing the value of any other digit in the number.



Let us break down a denary number and examine how each digit of a denary number and its *place value* contributes to the value of the overall number. Only naming of the ones and tens place has been included in the table below because of the amount of space naming takes up. Here we split the number 39705 into digits and place each digit into the appropriate box according to its ***place value***.

Table 1: *Denary Number Place Values*

place name		...				*tens*	*ones*
place value	**exponential**	10^5	10^4	10^3	10^2	10^1	10^0
	denary	100,000	10,000	1000	100	10	1
denary digits		0	3	9	7	0	5

We can write an equation that expresses the denary number broken down into the individual digits multiplied by that digit's place value.

(**0** * 100,000) + (**3** * 10,000) + (**9** * 1000) + (**7** * 100) + (**0** * 10) + (**5** * 1) = **39705**

For the original number, 39705, the *most significant digit* is the 3 in the *ten thousands* place, and it represents the value of thirty thousand (30,000). The *least significant digit* is 5, which is in the *ones* place, and represents a value of 5.

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1. Given the denary number 5013, answer the questions and complete the tables below.

a)

place value	exponential	10^5	10^4	10^3	10^2	10^1	10^0
	denary	100,000	10,000	1000	100	10	1
denary digits		0	0	5	0	1	3

b) Write an equation that expresses the denary number in the table from part (a) broken down into the individual digits multiplied by that digit's place value.

(**0** * 100,000) + (**0** * 10,000) + (**5** * 1000) + (**0** * 100) + (**1** * 10) + (**3** * 1) = **5013**

c) What is the *most significant digit* of the number 5013?	5
d) What value does this *most significant digit* represent?	(5 * 1000) = 5000
e) What is the *least significant digit* of the number 5013?	3
f) What value does this *least significant digit* represent?	(3 * 1) = 3

Binary Numbers: Bit Position

For *binary* (base-2) numbers, we don't have the same naming of place values. Instead, a ***bit position*** is used. The *bit position* is a number that starts with the right-most digit as bit zero (0), and increases with each digit going left. The table below shows bit positions of the binary number 11010110.

Table 2: *Binary Number Bit Positions*

bit position	7	6	5	4	3	2	1	0
binary digits (bits)	1	1	0	1	0	1	1	0

Binary Numbers: Place Value

Next, we reproduce a table essentially equivalent to *Table 1: Denary Number Place Values*, except to show the place values of binary digits. This table again contains the binary number 11010110.

Table 3: *Binary Number Place Values*

bit position		7	6	5	4	3	2	1	0
place value	**exponential**	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	denary	128	64	32	16	8	4	2	1
binary digits (bits)		1	1	0	1	0	1	1	0

As we showed for denary numbers, we can also write an equation that expresses the binary number broken down into the individual digits multiplied by that digit's place value.

(**1** * 128) + (**1** * 64) + (**0** * 32) + (**1** * 16) + (**0** * 8) + (**1** * 4) + (**1** * 2) + (**0** * 1)

= 128 + 64 + 16 + 8 + 4 + 2 = **214**

Since we're representing the *place value* in denary, by calculating the sum, we have converted from a binary number into a denary number. So the binary number 11010110 is equal to the denary number 214.

Binary Numbers: Bytes and Nibbles

Due to a combination of history and how binary numbers are stored in memory chips, when we are working with binary digits, it is most common to store binary numbers with a number of digits equal to a multiple of eight, usually one of: 8, 16, 32, 64, or 128. Very early on in the history of computing (in 1956), a group of 8 bits became known as a ***byte***. (And yes, it was a play on the word bite being a small morsel of food, with the change in spelling to prevent confusion with the word bit.) This also lead to the ***nibble*** referring to a group of 4 bits. (When eating, a nibble is smaller than a bite.)

Binary Numbers: Writing Binary Numbers

With denary numbers, when writing a long number, it is common to make the number more readable by separating digits of the number into groups of three using commas. For example, the speed of light in a vacuum is defined as 299792458 m/s – which is easier to read when written as 299,792,458 m/s.

For many computer languages, including Java, C#, and Python, binary numbers are preceded by the characters "0b" and can be made more readable by including an underscore between any two digits. For example, the denary number 214 can be written in those programming languages as the binary number: "**0b**1101_0110" (the bold has been added to highlight the prefix). Double underscores are invalid, as is an underscore immediately following the prefix, so these two representations are <u>not</u> valid:
✗"0b_1101_0110" and ✗"0b1101__0110".

Binary Numbers: Most and Least Significant Bits

When we write a denary number, it is rather unusual to write any ***leading zeros*** (zeros to the left of the first non-zero digit). For example, we would typically write 102 rather than 00102. Thus, when we write a denary number, the most significant digit is almost always non-zero.

Since binary numbers are typically stored with a multiple of 8 digits, it is very common for a written binary number to include a leading zero.

When writing a binary number, the ***most significant bit*** of the binary number is always the left-most bit of the number, while the ***least significant bit*** of the binary number is always the right-most bit. For the binary number in *Table 3: Binary Number Place Values*, the most significant bit is the digit 1 in bit position 7, which represents a value of 128 (calculated using the formula $1 * 2^7$), and the least significant bit is the 0 in position 0, which represents a value of 0 (calculated using the formula: $0 * 2^0$).

This is specifically when writing a binary number. Do not confuse this with transmission of binary data where either the most significant or the least significant bit might be transmitted first. This is a confusion we can deal with later when we discuss computer networking.

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2. Given the binary number 1010_0101, answer the questions and complete the tables below.

a)

bit position		7	6	5	4	3	2	1	0
place value	exponential	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	denary	128	64	32	16	8	4	2	1
binary digits (bits)		1	0	1	0	0	1	0	1

b) Write an equation that expresses the binary number in the table from part (a) broken down into the individual digits and multiplied by that digit's place value, then combine the values to convert the binary number into its denary number equivalent.

(1 * 128) + (0 * 64) + (1 * 32) + (0 * 16) + (0 * 8) + (1 * 4) + (0 * 2) + (1 * 1)

= 128 + 32 + 4 + 1 = **165**

c) What is the *most significant digit* of the binary number 1010_0101?	1
d) What value does this *most significant digit* represent?	(1*128) = 128
e) What is the *least significant digit* of the binary number 1010_0101?	1
f) What value does this *least significant digit* represent?	(1 * 1) = 1

3. Given the binary number 0111_1110, answer the questions and complete the tables below.

a)

bit position		7	6	5	4	3	2	1	0
place value	exponential	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	denary	128	64	32	16	8	4	2	1
binary digits (bits)		0	1	1	1	1	1	1	0

b) Write an equation that expresses the binary number in the table from part (a) broken down into the individual digits and multiplied by that digit's place value, then combine the values to convert the binary number into its denary number equivalent.

(0 * 128) + (1 * 64) + (1 * 32) + (1 * 16) + (1 * 8) + (1 * 4) + (1 * 2) + (0 * 1)

= 64 + 32 + 16 + 8 + 4 + 2 = **126**

c) What is the *most significant digit* of the binary number 1010_0101?	0
d) What value does this *most significant digit* represent?	(0*128) = 0
e) What is the *least significant digit* of the binary number 1010_0101?	0
f) What value does this *least significant digit* represent?	(0 * 1) = 0